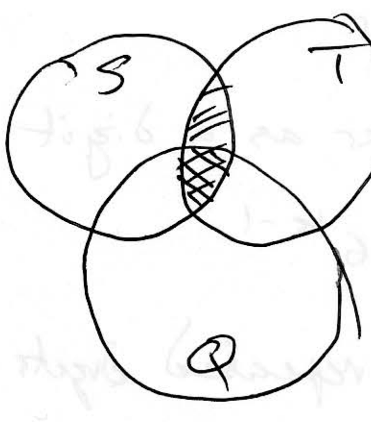
26 OCT 2015 Basic Counting Principles ~~Principles~~ Ted Codd

computer science

wk 5 - lectures 17-18

S & T are disjoint then see slide



Be careful to ~~keep~~ ensure the intersected areas are counted just once each.

$|S \cup T \cup Q| = |S| + |T| + |Q|$
$- |S \cap T| - |S \cap Q| - |Q \cap T|$
$+ |S \cap Q \cap T|$

how many pairs from 10 British, with each pair speaking different languages

$15 \times 10 = 150$

$10 \times 20 = 200$

15×20 third type $= 300$

overall 650 using sum rule.

EXAMPLE 1 PASSWORDS

password is 6, 7 or 8 digits/letters
each must contain at least 1 digit
how many if no character the same?

ignoring the "1 is a digit."
how many 6+7+8 length k

36^k (26 + 10 digits)

assuming 1 is a digit...

$10 \times 36^{k-1}$

digit   one less letter/number

but this locks first character as a digit

so perhaps: $k \times 10 \times 36^{k-1}$

BUT we need to exclude repeated digits eg catch22

see solution 1 slide!

But better, take total number of strings 36^k and exclude those without any digits 26^k

$\underline{36^k - 26^k}$

if all characters must differ:

There are 'factorial functions' ! to calculate the number of permutations / number of orders of different elements. $(n-1)!$ if tree in middle

$n!$ if tree position matters.

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k-permutations and arrangements

wk5

number of k permutations of set with n distinct elements:

$$P(n,k) = n*(n-1)*\dots*(n-k+1) = n!/(n-k)!$$

$$\frac{n!}{(n-k)!} = n\cdot(n-1)\cdot(n-2)*\dots*(n-k+1)$$

By theorem 1

by convention $0! = 1$

Ex 4: consider 10-size strings with 3 As, 5 Bs

In a string we cannot distinguish among identical letters

So 10! counts particular strings more than once.

We must exclude the internal orders of identical elements

$$\frac{10!}{(3!5!)}$$

$$\frac{n!}{(n-k)!}$$

Number of permutations of set of 'n' elements

q_1 elements of type 1

q_2 " " " 2

q_k " " " k

$n!/(q_1!$ (see slide)

$\boxed{3.3}$

$$\frac{n!}{(n-k)!\,k!} \qquad C_n^k \quad C(n,k)$$

see slide!

Binomial Coefficients

Combinations known as this because - see slide!

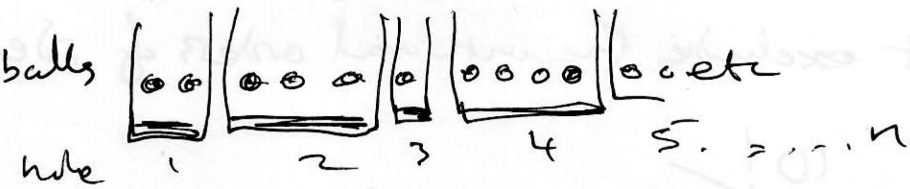
Combinations with repetitions.

how many k-combinations of set of n elements if those elements can repeat in the selection.

how many ways can we distribute k identical balls among n different holes?

= how many positive integer values are there for

$$\underbrace{x_1 + x_2 + \dots + x_n}_{\text{holes}} = k \, ?$$



$$\frac{(k+n-1)!}{k!(n-1)!} = \binom{\cancel{k}k+n-1}{k}$$

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wk5 How many groups of $4n$ students can be made, all with exactly 4 students?

$(4n)!$ is the number if each student has a task, etc. so that the order matters.

If the order of classes, and order of students doesn't matter:

4 people out — now group is 4 smaller — now 8 smaller

$$\binom{4n}{4}\binom{4n-4}{4}\binom{4n-8}{4}\dots$$

Football 8 games. At most 7 goals.

result $x : y$ so $x + y \leq 7$ goals

two different teams so $y : x$ also relevent.

2 different teams.

$$\binom{7+3-1}{2} = \frac{9!}{7!\,2!} = \underline{36}$$

a way to consider 3 gates = each goal + missed

36^8 for 8 games