COMP 1215 Foundations of Computer Science
9 NOV 2015 wk 7
lectures 23 + 24

Jed Gibbs for Charlton Rodda

* Class test for Thursday first session
(NB this will be in the next test along with probability After That)

A powerful technique to prove statements of this kind:

suppose we have a set X with 4 elements we would prove each element is divisible by 2 - but with a larger set that approach is impractical.

Generalization:

$x = \{6n \mid n \in \mathbb{N}\}$ is the set "multiples of 6"

since any x can be written as $2 \times 3n$ that proves that all x are divisible by 2.

Induction Principle

If we can show statement true for $k > 1$ then we can prove also true for $k+1$

because $k > 1$ so $k+1 > 2$

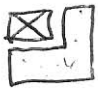
actually proves true for all n natural numbers.

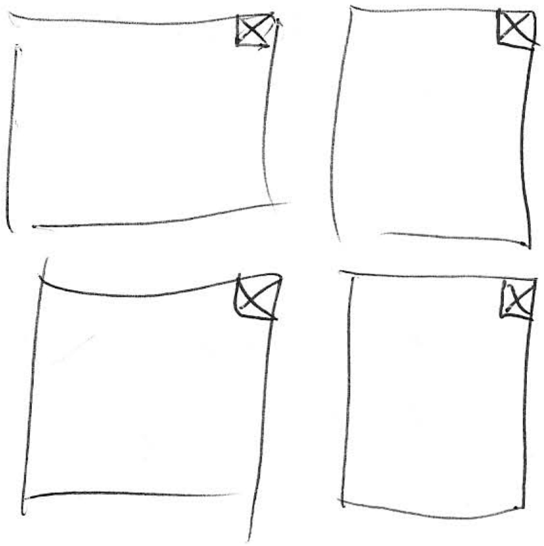
if we can prove statements above the line base case + step case

then we can conclude the statement below the line

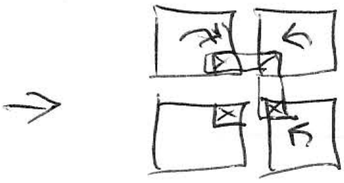
Example 2

shaped tiles. fill a square with these tiles leaving one x one square in the near ~~centre~~ centre for a picture:





rotate to make  in centre:



let P(n) be 'we will fill a grid of size 2k x 2k then rotate 3

Why is proof by induction valid?

Property of natural numbers is every natural number is reachable by the operation "add 1"

only valid for sets with initial base points

sets must be wellordered - each of the non-empty subsets has to have a least element.

holds for naturals but not reals.

It is necessary to show the base case.

P(k) ~~That is~~

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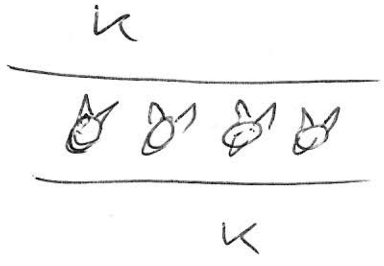
false proof: P(k) any set of horses are of same colour

take 2 subsets of size k, ~~differ~~ differing by 1 element.
trying to prove all horses the same colour (see slide)



The idea that 2 sets overlap
but for k = 1 there is no intersection

so the 2 chosen subsets do not necessarily overlap.

Common Knowledge Puzzle – 'dirty faces'

$N \geq 1$ children play; $K \geq 1$ get dirty faces.
children stand in circle, can see others' faces dirty
- adult asks repeatly that children with dirty faces raise their hands. On the kth time all children with dirty faces raise their hands.

Induction

if only 1 child base case k = 1 there is only 1 child – if someone has a dirty face it must be this child.

If k = 2 the 2 children can see the other. There may be one, maybe 2.

For k dirty faces all k will raise hand on kth round and not before

Add a further child $(k+1)$ if at kth round the others have not yet raised their hands, then I must have a dirty face too and all will raise in the kth $+1$

$\boxed{\text{On board:}}$

$n^3 + 2n \qquad n \in N_4$

Base $\quad n = 1 \qquad 1^3 + 2\,1 = 1 + 2 = 3$ ✓

Induction $\qquad P(1) \qquad P(k) \quad k^2 + 2k = 3\underline{m}$

step $\qquad P(k+1) \quad (k+1)^3 + 2(k+1)$

$$= k^3 + 3k^2 + 3k + 1 + 2k + 2$$

$$= [k^3 + 2k] + 3(k^2 + k + 1)$$

$$= 3m + 3(k^2 + k + 1)$$

$$= 3(m + k^2 + k + 1) \checkmark$$

Don't need to write much more than this but should resolve to the point that the result can be put into a calculator to give result.

Recommend to reread proof as Pascal

$$\binom{n+1}{k} = \underbrace{\binom{n}{k}}_{\text{not present}} + \underbrace{\binom{n}{k-1}}_{\text{present}}$$

? hard to read on the board!

Proving every positive integer can be written as a product of prime numbers...

Base 2 prime, 3 prime, $4 = 2 \times 2$, 5 prime, $6 = 3 \times 2$

$k \geq 7$

$P(k+1)$

& if $k+1$ is prime ✓

or

$P(k+1) - P(k)$

assuming multiple

$1 < a, b < k+1$

$a_1 \times a_2$ $\quad b_1 \times b_2$

if $k+1$ not prime then

$k+1 = a \times b$